

**SECOND SEMESTER EXAMINATION, 2008-09**

**MATHEMATICS-II**

(EAS-203)

Time : 3 Hours]

[Total Marks : 100

**SECTION-A**

**Note :** (1) Attempt all questions. All questions carry equal marks.

Fill up the appropriate answers in the space provided :

1. (a) The differential equation

$$\left(\frac{d^3y}{dx^3}\right)^4 - 6x^2\left(\frac{dy}{dx}\right)^2 + e^x = \sin xy \text{ is of ..... order and ..... degree.}$$

Ans. order = 3, degree = 4

(b) A particular solution of the differential equation  $\frac{dy}{dt} = \frac{1}{2}(y^2 - 1)$  that satisfies the initial condition  $y(0) = 2$  is .....

Ans.  $3(y - 1) = e^t(y + 1)$

(c) The general solution of the differential equation  $\frac{d^5y}{dx^5} - \frac{d^3y}{dx^3} = 0$  is given by .....

Ans.  $y = (c_1 + c_2x + c_3x^2) + c_4e^x + c_5e^{-x}$

Pick up the Correct Answer from the following :

(d) The Rodrigues formula for Legendre Polynomial  $P_n(x)$  is given by

(i)  $P_n(x) = \frac{1}{n! 2^n} \frac{d^n}{dx^n} (x^2 - 1)^n$

(ii)  $P_n(x) = \frac{n!}{2^n} \frac{d^n}{dx^n} (x^2 - 1)^{n-1}$

(iii)  $P_n(x) = \frac{n!}{2^{n-1}} \frac{d^n}{dx^n} (x^2 - 1)^{n-1}$

(iv)  $P_n(x) = \frac{1}{n! 2^n} (x^2 - 1)^n$

Ans. (i)  $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$

(e) The Laplace transform of the function

$$f(t) = \begin{cases} 1, & 0 \leq t < 2 \\ -1, & 2 \leq t < 4, f(t+4) = f(t) \end{cases} \text{ is given as}$$

(i)  $\frac{1 - e^{-2s}}{s(1 + e^{-2s})}$

$$(ii) \frac{1 + e^{-2s}}{s(1 + e^{-2s})} \quad (iii) 0 \quad (iv) \frac{s+1}{s-1}$$

$$\text{Ans. (i) } \frac{1 - e^{-2s}}{s(1 + e^{-2s})}$$

(f) The inverse Laplace transform of  $\log\left(\frac{s+1}{s-1}\right)$  is given by

$$(i) \frac{2}{t} \cosh t \quad (ii) \frac{2}{t} \sinh t \quad (iii) 2t \cos t \quad (iv) 2t \sin t$$

$$\text{Ans. (ii) } \frac{2}{t} \sinh t$$

Indicate True/False for the statements made therein -

(g) (i) A function  $f(x)$  is even if  $f(-x) = -f(x)$ . (True/False)

(ii) A function  $f(x)$  is odd if  $f(-x) = f(x)$ . (True/False)

(iii) Most functions are neither even nor odd.

(True / False)

(iv) A function  $f(x)$  can always be expressed as an arithmetic mean of an even and odd function as

$$f(x) = \frac{1}{2} [f(x) + f(-x)] + \frac{1}{2} [f(x) - f(-x)]$$

(True/False)

Ans. (i) False (ii) False (iii) True (iv) True.

(h) (i) With usual symbols, the PDE

$u_{xx} + u^2 u_{yy} = f(xy)$  is non-linear in 'u' and is of second order. (True/False)

(ii) The small transverse vibrations of a string are governed by one dimensional heat equation  $y_t = a^2 y_{xx}$ . (True/False)

(iii) Two dimensional steady state heat flow is given by Laplace's equation  $t = a^2(u_{xx} + u_{tt})$ . (True/False)

(iv) The PDE of all sphere whose centre lie on z-axis and given by equations  $z^2 + y^2 + (z-a)^2 = b^2$ , a and b being constants, are governed by  $xz_y - yz_x = 0$ . (True/False)

Ans. (i) True (ii) False (iii) False (iv) True.

(i) Applying the method of separation of variables techniques, the solution to the P.D.E.  $3u_x + 2u_y = 0$  is ....., where  $u_x = \frac{\partial u}{\partial x}$ ,  $u_y = \frac{\partial u}{\partial y}$ .

$$\text{Ans. } c_1 c_2 e^{hx/3} e^{-hy/2}$$

Match the column for the items of the left side to that of right side :

(j) A second order P.D.E. in the function 'u' of two independent variables x, y with usual symbols

$Au_{xx} + Bu_{xy} + Cu_{yy} + F(u) = 0$ , then

(i) Hyperbolic (a)  $B^2 - 4AC = 0$

(ii) Parabolic (b)  $B^2 - 4AC < 0$

(iii) Elliptic (c)  $B^2 - 4AC > 0$

(iv) Not classified (d)  $A = B = C = 0$

Ans. (i) (b), (ii) (a), (iii) (c) (iv) (d).

## SECTION -B

**Note :** Attempt any three questions from this section. all questions carry equal marks. 3 × 10 = 30

2. (a) Apply the method of variation of parameters to solve the ordinary differential equations

$$\frac{d^2 y}{dx^2} + y = \tan x, \quad 0 < x < \pi/2$$

Ans. Here  $R = \tan x$

A.E. is  $m^2 + 1 = 0 \Rightarrow m = \pm i$

C.F. =  $c_1 \cos x + c_2 \sin x$

Let  $y_1 = \cos x$ ,  $y_2 = \sin x$

$y_1' = -\sin x$ ,  $y_2' = \cos x$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$\begin{aligned} \text{P.I.} &= -y_1 \int \frac{y_2 R}{w} dx + y_2 \int \frac{y_1 R}{w} dx \\ &= -\cos x \int \frac{\sin x \cdot \tan x}{1} dx + \sin x \int \frac{\cos x \cdot \tan x}{1} dx = -\cos x \int \frac{\sin^2 x}{\cos x} dx + \sin x \int \sin x dx \\ &= -\cos x \int \left( \frac{1 - \cos^2 x}{\cos x} \right) dx + \sin x (-\cos x) = -\cos x \int (\sec x - \cos x) dx - \sin x \cos x \\ &= -\cos x [\log(\sec x + \tan x) - \sin x] - \sin x \cos x = -\cos x \log(\sec x + \tan x). \end{aligned}$$

The complete solution is

$y = \text{C.F.} + \text{P.I.}$

$y = c_1 \cos x + c_2 \sin x - \cos x \log(\sec x + \tan x).$

**Q. 2. (b) Show that the Bessel's function  $J_n(x)$  is an even function when  $n$  is even and is odd function when  $n$  is odd. Express  $J_6(x)$  in terms of  $J_0(x)$  and  $J_1(x)$ .**

Ans. Suppose  $n$  is even

$$J_n(-x) = (-x)^n \sum_{m=0}^{\infty} \frac{(-1)^m (-x)^{2m}}{2^{2m+n} \cdot m! (m+n)!}$$

for  $n$  even  $(-1)^n = 1$  and  $(-1)^{2m} = 1$

$$J_{-n}(-x) = x^n \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+n} \cdot m! (m+n)!} = J_n(x)$$

Thus  $J_n(x)$  is an even function.

Suppose  $n$  is odd. Then  $(-1)^n = -1$ .

$$J_n(-x) = -x^n \sum_{m=0}^{\infty} \frac{(-1)^n x^{2m}}{2^{2m+n} m!(m+n)!} = -J_n(x).$$

This  $J_n(x)$  is an odd function.

By the recurrence relation

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x) \quad \dots(1)$$

Put  $n = 1, 2, 3, 4, 5$ , in (1), we get

$$J_2 = \frac{2}{x} J_1(x) - J_0(x) \quad \dots(2)$$

$$J_3 = \frac{4}{x} J_2(x) - J_1(x) \quad \dots(3)$$

$$J_4 = \frac{6}{x} J_3(x) - J_2(x) \quad \dots(4)$$

$$J_5 = \frac{8}{x} J_4(x) - J_3(x) \quad \dots(5)$$

$$J_6 = \frac{10}{x} J_5(x) - J_4(x) \quad \dots(6)$$

Substituting (3) in (2)

$$J_3 = \left( \frac{8}{x^2} - 1 \right) J_1 - \frac{4}{x} J_0 \quad \dots(7)$$

Substituting (7) and (2) in (4),

$$J_4 = \left( \frac{48}{x^3} - \frac{8}{x} \right) J_1 + \left( 1 - \frac{24}{x^2} \right) J_0 \quad \dots(8)$$

Substituting (8) and (3) in (5)

$$J_5 = \left( \frac{384}{x^4} - \frac{72}{x^2} - 1 \right) J_1 + \left( \frac{12}{x} - \frac{192}{x^3} \right) J_0 \quad \dots(9)$$

Substituting (9) and (8) in (6), we get

$$J_6(x) = \frac{10}{x} \left[ \left( \frac{384}{x^4} - \frac{72}{x^2} - 1 \right) J_1 + \left( \frac{12}{x} - \frac{192}{x^3} \right) J_0 \right] - \left( \frac{48}{x^3} - \frac{8}{x} \right) J_1 - \left( 1 - \frac{24}{x^2} \right) J_0$$

$$J_6(x) = \left( \frac{3840}{x^4} - \frac{768}{x^3} - \frac{2}{x} \right) J_1(x) + \left( \frac{144}{x^2} - 1 - \frac{1920}{x^4} \right) J_0(x).$$

**Q. 2. (c) Use convolution theorem to find the inverse of the function  $\frac{1}{(s^2 + a^2)^2}$ .**

**Ans.** Let  $f(s) = \frac{1}{(s^2 + a^2)}$ ,  $g(s) = \frac{1}{(s^2 + a^2)}$

$$L^{-1}\{f(s)\} = \frac{1}{a} \sin at = F(t), \quad L^{-1}\{g(s)\} = \frac{1}{a} \sin at = G(t)$$

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By convolution theorem

$$\begin{aligned}
 L^{-1}\{f(s)g(s)\} &= \int_0^t F(x)G(t-x)dx \\
 &= \int_0^t \frac{1}{a} \sin ax \cdot \frac{1}{a} \sin(at-ax) dx = \frac{1}{2a^2} \int_0^t 2 \sin ax \sin(at-ax) dx \\
 &= \frac{1}{2a^2} \int_0^t [\cos(ax-at+ax) - \cos(ax+at-ax)] dx = \frac{1}{2a^2} \int_0^t [\cos(2ax-at) - \cos at] dx \\
 &= \frac{1}{2a^2} \left[ \frac{\sin(2ax-at)}{2a} - x \cos at \right]_0^t = \frac{1}{2a^2} \left[ \left( \frac{1}{2a} \sin at - t \cos at \right) - \left( -\frac{1}{2a} \sin at \right) \right] \\
 &= \frac{1}{2a^2} \left[ \frac{1}{a} \sin at - t \cos at \right] = \frac{1}{2a^3} (\sin at - at \cos at)
 \end{aligned}$$

**Q. 2. (d) Obtain the Fourier series of  $f(x) = \left(\frac{\pi-x}{2}\right)$  in the interval  $(0, 2\pi)$  and hence**

**deduce**  $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

**Ans.**  $f(x) = \left(\frac{\pi-x}{2}\right)$

Fourier series of  $f(x)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \dots(1)$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} \left(\frac{\pi-x}{2}\right) dx = \frac{1}{2\pi} \left[ \pi x - \frac{x^2}{2} \right]_0^{2\pi} = \frac{1}{2\pi} \{ (2\pi^2 - 2\pi^2) - (0 - 0) \} = 0$$

$$\begin{aligned}
 a_n &= \frac{1}{\pi} \int_0^{2\pi} \left(\frac{\pi-x}{2}\right) \cos nx dx = \frac{1}{2\pi} \left[ (\pi-x) \left( \frac{\sin nx}{n} \right) - (-1) \left( \frac{-\cos nx}{n^2} \right) \right]_0^{2\pi} \\
 &= \frac{1}{2\pi} \left[ \left\{ (\pi-2\pi) \left( \frac{\sin 2n\pi}{n} \right) - \frac{\cos 2n\pi}{n^2} \right\} - \left\{ 0 - \frac{1}{n^2} \right\} \right] = \frac{1}{2\pi} \left[ -\frac{(-1)^{2n}}{n^2} + \frac{1}{n^2} \right] = \frac{1}{2\pi} \left[ -\frac{1}{n^2} + \frac{1}{n^2} \right] = 0
 \end{aligned}$$

$$\begin{aligned}
 b_n &= \frac{1}{\pi} \int_0^{2\pi} \left(\frac{\pi-x}{2}\right) \sin nx dx = \frac{1}{2\pi} \left[ (\pi-x) \left( \frac{-\cos nx}{n} \right) - (-1) \left( \frac{-\sin nx}{n^2} \right) \right]_0^{2\pi} \\
 &= \frac{1}{2\pi} \left[ \left\{ -(\pi-2\pi) \frac{\cos 2n\pi}{n} - 0 \right\} - \left( \frac{-\pi}{n} \right) \right] = \frac{1}{2\pi} \left[ \frac{\pi(-1)^{2n}}{n} + \frac{\pi}{n} \right] = \frac{1}{2\pi} \left[ \frac{\pi}{n} + \frac{\pi}{n} \right] = \frac{1}{n}
 \end{aligned}$$

Hence the required fourier series is

$$\left(\frac{\pi-x}{2}\right) = \sum_{n=1}^{\infty} \left(\frac{1}{n}\right) \sin nx$$

$$\frac{\pi-x}{2} = \sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \frac{\sin 4x}{4} + \frac{\sin 5x}{5} + \dots$$

$$\text{put } x = \frac{\pi}{2}$$

$$\frac{\pi}{4} = \sin \frac{\pi}{2} + \frac{\sin \pi}{2} + \frac{\sin \frac{3\pi}{2}}{3} + \frac{\sin 2\pi}{4} + \frac{\sin \frac{5\pi}{2}}{5} + \dots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} + \dots$$

(e) A string of Length 'L' is stretched and fastened to two fixed points, Find the solution of the wave equation  $y_{tt} = a^2 y_{xx}$  when initial displacement is  $y(x, 0) = f(x) = b \sin\left(\frac{\pi x}{L}\right)$ , where symbols have usual meaning.

$$\text{Ans. Given wave equation } y_{tt} = a^2 y_{xx} \quad \dots(1)$$

$$\text{Initial conditions } y(x, 0) = f(x) = b \sin\left(\frac{\pi x}{L}\right) \quad \dots(2)$$

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0 \quad \dots(3)$$

Boundary conditions

$$y(0, t) = 0 \quad \dots(4)$$

$$y(L, t) = 0 \quad \dots(5)$$

Let  $y = X(x)T(t)$  be the solution of eq<sup>n</sup> (1)

$$(1) \Rightarrow \frac{T''}{a^2 T} = \frac{X''}{X} = \text{const.}$$

When const =  $-p^2$

$$\frac{X''}{X} = -p^2$$

$$X'' = -p^2 X$$

$$X'' + p^2 X = 0$$

$$m^2 + p^2 = 0 \Rightarrow m = \pm pi$$

$$X = c_1 \cos px + c_2 \sin px$$

$$y(x, t) = (c_1 \cos px + c_2 \sin px) (c_3 \cos apt + c_4 \sin apt) \quad \dots(6)$$

using B.C. (4),  $y(0, t) = 0$  in (6), we get

$$0 = c_1(c_3 \cos apt + c_4 \sin apt)$$

$$\Rightarrow \boxed{c_1 = 0}$$

using B.C. (5),  $y(L, t) = 0$  in (6) we get

$$0 = (c_1 \cos as. pL + c_2 \sin pL) (c_3 \cos apt + c_4 \sin apt)$$

$$\Rightarrow c_2 \sin pL = 0 \Rightarrow \sin pL = 0 = \sin n\pi$$

$$\boxed{p = \frac{n\pi}{L}}$$

$$(6) \Rightarrow y(x, t) = c_2 \sin \frac{n\pi x}{L} \left( c_3 \cos \frac{n\pi at}{L} + c_4 \sin \frac{n\pi at}{L} \right) \quad \dots(7)$$

$$\frac{\partial y}{\partial t} = c_2 \sin \frac{n\pi x}{L} \left( -c_3 \sin \frac{n\pi at}{L} + c_4 \cos \frac{n\pi at}{L} \right) \left( \frac{n\pi a}{L} \right)$$

using I.C. (3),  $\left( \frac{\partial y}{\partial t} \right)_{t=0} = 0$

$$0 = c_2 \sin \frac{n\pi x}{L} (-0 + c_4) \left( \frac{n\pi a}{L} \right)$$

$$\Rightarrow \boxed{c_4 = 0}$$

$$(7) \Rightarrow y(x, t) = c_2 c_3 \sin \left( \frac{n\pi x}{L} \right) \cos \left( \frac{n\pi at}{L} \right)$$

...(8)

using I.C. (2),  $y(x, 0) = f(x) = b \sin \left( \frac{\pi x}{L} \right)$  in (8), we get

$$b \sin \left( \frac{\pi x}{L} \right) = c_2 c_3 \sin \left( \frac{n\pi x}{L} \right)$$

$$\Rightarrow c_2 c_3 = b \text{ and } n = 1.$$

Hence the required solution is

$$y(x, t) = b \sin \left( \frac{\pi x}{L} \right) \cos \left( \frac{\pi at}{L} \right)$$

### SECTION - C

**Note :** Attempt all questions from this section, selecting any two parts from each question. All questions carry equal marks.

[5 × 2] × 5 = 50

**Q. 3. (a)** The equations of motion of a particle are given by  $\frac{dx}{dt} + wy = 0$ ,  $\frac{dy}{dt} - wx = 0$

**Find the path of the particle and show that it is a circle.**

**Ans.** The equation of motion

$$\frac{dx}{dt} + wy = 0$$

$$\frac{dy}{dt} - wx = 0$$

or  $Dx + wy = 0$

...(1)

$$wx + Dy = 0$$

...(2)

Eliminating  $y$  in (1) & (2), by multiplying (1) by  $D$  and (2) by  $w$  and subtracting, we get

$$(D^2 + w^2)x = 0$$

$$\text{A.E. is } m^2 + w^2 = 0 \Rightarrow m = \pm iw$$

$$x = c_1 \cos wt + c_2 \sin wt$$

...(3)

$$\frac{dx}{dt} = -c_1 w \sin wt + c_2 w \cos wt.$$

$$\text{from (1), } y = -\frac{1}{w} (Dx) = -\frac{1}{w} (-c_1 w \sin wt + c_2 w \cos wt)$$

$$y = c_1 \sin wt - c_2 \cos wt$$

...(4)

squaring and adding (3) & (4), we get

$$x^2 + y^2 = c_1^2 + c_2^2 = \text{constant}$$

Hence the path of the particle is a circle.

**Q. 3. (b) Integrate the differential equation**

$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + w^2x = a \cos pt$  and give the physical interpretation of the complete

solution. Also deduce that the solution takes the form

$$x = \frac{a \cos(pt - \theta)}{\sqrt{(w^2 - p^2)^2 + 4k^2p^2}} \left( \tan \theta = \frac{2kp}{w^2 - p^2} \right) \text{ As } t \rightarrow \infty$$

Ans.  $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + w^2x = a \cos pt$

A.E. is  $m^2 + 2km + w^2 = 0$

$$m = -k \pm \sqrt{k^2 - w^2}$$

$$\text{C.F.} = e^{-kt} \left[ c_1 e^{t\sqrt{k^2 - w^2}} + c_2 e^{-t\sqrt{k^2 - w^2}} \right]$$

It represents the free oscillations of the system which die out as  $t \rightarrow \infty$

$$\begin{aligned} \text{P.I.} &= \frac{1}{D^2 + 2kD + w^2} a \cos pt = a \frac{1}{-p^2 + 2kD + w^2} \cos pt \\ &= a \cdot \frac{[(w^2 - p^2) - 2kD]}{(w^2 - p^2)^2 - 4k^2D^2} \cos pt = a \frac{[(w^2 - p^2) - 2kD]}{(w^2 - p^2)^2 + 4k^2p^2} \cos pt \\ &= \frac{a}{(w^2 - p^2)^2 + 4k^2p^2} [(w^2 - p^2) \cos pt + 2kp \sin pt] \end{aligned}$$

putting  $R \cos \theta = (w^2 - p^2)$ ,  $R \sin \theta = 2kp$

$$R = \sqrt{(w^2 - p^2)^2 + 4k^2p^2}, \quad \tan \theta = \frac{2kp}{w^2 - p^2}$$

$$= \frac{a}{(w^2 - p^2)^2 + 4k^2p^2} R \cos(pt - \theta) = \frac{a \sqrt{(w^2 - p^2)^2 + 4k^2p^2}}{(w^2 - p^2)^2 + 4k^2p^2} \cos(pt - \theta)$$

$$\text{P.I.} = \frac{a \cos(pt - \theta)}{\sqrt{(w^2 - p^2)^2 + 4k^2p^2}}, \text{ where } \tan \theta = \frac{2kp}{w^2 - p^2}$$

Which represents the forced oscillation of the system having

(i) constant amplitude  $= \frac{a}{\sqrt{(w^2 - p^2)^2 + 4k^2p^2}}$

(ii) period  $= \frac{2\pi}{p}$

The complete solution is

$$x = \text{C.F.} + \text{P.I.}$$

$$x = e^{-kt} \left[ c_1 e^{t\sqrt{k^2 - w^2}} + c_2 e^{-t\sqrt{k^2 - w^2}} \right] + \frac{a \cos(pt - \theta)}{\sqrt{(w^2 - p^2)^2 + 4k^2p^2}}$$



when  $t \rightarrow \infty$ , the free oscillation (C.F.) die away while the forced oscillations continue giving the steady state motion. Then

$$x = \frac{a \cos(pt - \theta)}{\sqrt{(w^2 - p^2)^2 + 4k^2 p^2}} \text{ where } \tan \theta = \frac{2kp}{w^2 - p^2}$$

**Q. 3. (c) Find the complete solution of the differential equation**  
 $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = xe^x \cos x.$

**Ans.** A.E. is  $m^2 - 2m + 1 = 0$ ,  $(m - 1)^2 = 0 \Rightarrow m = 1, 1$

C.F. =  $(c_1 + c_2 x) e^x$

P.I. =  $\frac{1}{D^2 - 2D + 1} e^x x \cos x = \frac{1}{(D - 1)^2} e^x (x \cos x) = e^x \frac{1}{(D + 1 - 1)^2} x \cos x$

$$= e^x \frac{1}{D^2} x \cos x = e^x \frac{1}{D} \int x \cos x dx = e^x \frac{1}{D} [x(\sin x) - (-\cos x)] = e^x \int (x \sin x + \cos x) dx$$

$$= e^x [x(-\cos x) - (-\sin x) + \sin x] = -e^x [-x \cos x + 2 \sin x] = e^x (x \cos x - 2 \sin x)$$

The complete solution is

$y = \text{C.F.} + \text{P.I.}, y = (c_1 + c_2 x) e^x + e^x (x \cos x - 2 \sin x)$

**Q. 4 (a) For the Bessel's function, prove that**

$$J_{\frac{1}{2}}(x) = \left( \sqrt{\frac{2}{\pi x}} \right) \sin x.$$

**Ans.**  $J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r! (n+r+1)!} \left( \frac{x}{2} \right)^{n+2r}$

$$J_n(x) = \left( \frac{x}{2} \right)^n \left[ \frac{1}{n! (n+1)!} - \frac{1}{1! (n+2)!} \left( \frac{x}{2} \right)^2 + \frac{1}{2! (n+3)!} \left( \frac{x}{2} \right)^4 - \dots \right]$$

put  $n = \frac{1}{2}$

$$J_{1/2}(x) = \left( \frac{x}{2} \right)^{1/2} \left[ \frac{1}{\frac{1}{2}! \frac{3}{2}!} - \frac{1}{1! \frac{5}{2}!} \left( \frac{x}{2} \right)^2 + \frac{1}{2! \frac{7}{2}!} \left( \frac{x}{2} \right)^4 - \dots \right]$$

$$= \left( \frac{x}{2} \right)^{1/2} \left[ \frac{1}{\frac{1}{2}! \frac{3}{2}!} - \frac{1}{\frac{3}{2}! \frac{5}{2}!} \left( \frac{x}{2} \right)^2 + \frac{1}{2! \frac{5}{2}! \frac{7}{2}!} \left( \frac{x}{2} \right)^4 - \dots \right] = \frac{\sqrt{x}}{\sqrt{2} \frac{1}{2}!} \left[ \frac{2}{1!} - \frac{2x^2}{3!} + \frac{2x^4}{5!} - \dots \right]$$

Now multiplying the series by  $x/2$  and outside by  $\frac{2}{x}$ , we get

$$J_{1/2}(x) = \frac{\sqrt{2}}{\sqrt{x} \sqrt{\pi}} \left[ \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right] = \sqrt{\frac{2}{\pi x}} \sin x$$

$$x P_n'(x) = n P_n(x) + P_{n-1}'(x)$$

$$\text{Ans. } (1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x) t^n \quad \dots(1)$$

Differentiating (1) partially w.r.t  $x$ , we get

$$-\frac{1}{2}(1 - 2xt + t^2)^{-3/2}(-2t) = \sum_{n=0}^{\infty} P_n'(x) t^n, \quad t(1 - 2xt + t^2)^{-3/2} = \sum_{n=0}^{\infty} P_n'(x) t^n \quad \dots(2)$$

Again differentiating (1) partially w.r to  $t$ , we get

$$(x - t)(1 - 2xt + t^2)^{-3/2} = \sum_{n=0}^{\infty} n P_n(x) t^{n-1} \quad \dots(3)$$

Dividing (3) by (2), we get

$$\frac{x - t}{t} = \frac{\sum n P_n(x) t^{n-1}}{\sum P_n'(x) t^n}$$

$$\sum n P_n(x) t^n = (x - t) \sum P_n'(x) t^n$$

Equating coefficients of  $t^n$  from both sides, we get

$$n P_n(x) = x P_n'(x) - P_{n-1}'(x)$$

$$\text{or } x P_n'(x) = n P_n(x) + P_{n-1}'(x)$$

**Q. 4. (c) Express the polynomial  $f(x) = 4x^3 - 2x^2 - 3x + 8$  in terms of Legendre polynomials.**

$$\text{Ans. } f(x) = 4x^3 - 2x^2 - 3x + 8 = a P_3(x) + b P_2(x) + c P_1(x) + d P_0(x)$$

$$4x^3 - 2x^2 - 3x + 8 = a \left( \frac{5x^3 - 3x}{2} \right) + b \left( \frac{3x^2 - 1}{2} \right) + c(x) + d(1)$$

On comparing the coefficient of various power of  $x$ ,

$$\frac{5a}{2} = 4 \Rightarrow a = \frac{8}{5}$$

$$\frac{3b}{2} = -2 \Rightarrow b = -\frac{4}{3}$$

$$-\frac{3a}{2} + c = -3 \Rightarrow c = -3 + \frac{3a}{2} = -3 + \frac{3}{2} \left( \frac{8}{5} \right) = \frac{-3}{5}$$

$$-\frac{b}{2} + d = 8 \Rightarrow d = 8 + \frac{b}{2} = 8 + \frac{1}{2} \left( -\frac{4}{3} \right) = \frac{22}{3}$$

Hence

$$f(x) = \frac{8}{5} P_3(x) - \frac{4}{3} P_2(x) - \frac{3}{5} P_1(x) + \frac{22}{3} P_0(x).$$

**Q. 5. (a) Using Laplace transformation, show that  $\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$ .**

$$\text{Ans. Let } f(s) = \sin s, \quad \mathcal{L}\{f(s)\} = \mathcal{L}\{\sin s\} = \frac{1}{s^2 + 1} = F(s)$$

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s) ds, \quad L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \left(\frac{1}{s^2 + 1}\right) ds = [\tan^{-1} s]_s^\infty = \frac{\pi}{2} - \tan^{-1} s$$

$$\int_0^\infty \frac{\sin t}{t} dt = \int_0^\infty e^{-0t} \left(\frac{\sin t}{t}\right) dt = L\left\{\frac{\sin t}{t}\right\}, \text{ when } s = 0 = \left[\frac{\pi}{2} - \tan^{-1} s\right]_{s=0} = \frac{\pi}{2} - \tan^{-1}(0) = \frac{\pi}{2}.$$

**Q. 5. (b) A particle moves in a line so that its displacement  $x$  from a fixed point at any time  $t$ , is given by  $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 80 \sin 5t$ . Using Laplace transform, find displacement at any time  $t$  if initially particle is at rest at  $x = 0$ .**

**Ans.**  $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 80 \sin 5t$

$x(0) = 0, x'(0) = 0$

Taking Laplace transform of (1) we get

$[s^2 L(x) - sx(0) - x'(0)] + 4[sL(x) - x(0)] + 5L(x) = 80L(\sin 5t)$

$(s^2 + 4s + 5)L(x) = 80\left(\frac{5}{s^2 + 25}\right), \quad L(x) = 400\left[\frac{1}{(s^2 + 25)(s^2 + 4s + 5)}\right]$

on inversion,

$x = 400 L^{-1}\left[\frac{1}{(s^2 + 25)(s^2 + 4s + 5)}\right]$

$\frac{1}{(s^2 + 25)(s^2 + 4s + 5)} = \frac{As + B}{s^2 + 25} + \frac{Cs + D}{s^2 + 4s + 5}$

$1 = (As + B)(s^2 + 4s + 5) + (Cs + D)(s^2 + 25)$

$A + C = 0 \Rightarrow C = -A$

$B + D + 4A = 0$

$4B + 5A + 25C = 0$

$5B + 25D = 1$

on solving, we get

$B = -\frac{1}{40}, D = \frac{9}{200}, A = -\frac{1}{8}, C = \frac{1}{8}$

$\frac{1}{(s^2 + 25)(s^2 + 4s + 5)} = \frac{\left(-\frac{1}{8}s - \frac{1}{40}\right)}{s^2 + 25} + \frac{\left(\frac{1}{8}s + \frac{9}{200}\right)}{s^2 + 4s + 5}$

$x = 400 L^{-1}\left[-\frac{1}{40}\left(\frac{5s + 1}{s^2 + 25}\right) + \frac{1}{200}\left(\frac{25s + 9}{s^2 + 4s + 5}\right)\right]$

$= \frac{400}{200}\left[-5L^{-1}\left(\frac{5s}{s^2 + 25}\right) - 5L^{-1}\left(\frac{1}{s^2 + 25}\right) + L^{-1}\left(\frac{25s + 9}{s^2 + 4s + 5}\right)\right]$

$= -50 \cos 5t - \frac{10}{5} \sin 5t + 2L^{-1}\left[\frac{25(s + 2) - 50 + 9}{(s + 2)^2 + 1}\right]$

$$= -50 \cos 5t - 2 \sin 5t + 2e^{-2t} L^{-1} \left[ \frac{25s - 41}{s^2 + 1} \right]$$

$$x = -50 \cos 5t - 2 \sin 5t + 2e^{-2t} (25 \cos t - 41 \sin t)$$

**Q. 5. (c) Find the Laplace transform of the function**

$$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$$

$$\text{Ans. } f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$$

$$f(t) = (t-1)[u(t-1) - u(t-2)] + (3-t)[u(t-2) - u(t-3)]$$

$$f(t) = (t-1)u(t-1) - (t-1-3+t)u(t-2) + (t-3)u(t-3)$$

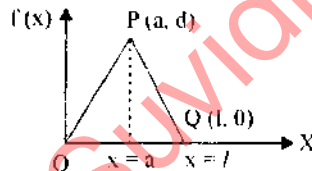
$$f(t) = (t-1)u(t-1) - 2(t-2)u(t-2) + (t-3)u(t-3)$$

Taking Laplace transform, we get,

$$L\{f(t)\} = L\{(t-1)u(t-1)\} - 2L\{(t-2)u(t-2)\} + L\{(t-3)u(t-3)\}$$

$$= e^{-s}L(t) - 2e^{-2s}L(t) + e^{-3s}L(t) = \frac{e^{-s}}{s^2} - \frac{2e^{-2s}}{s^2} + \frac{e^{-3s}}{s^2}$$

**Q. 6. (a) Find the half period sine series for  $f(x)$  given in the range  $(0, l)$  by the graph  $OPQ$  as shown in figure.**



$$\text{Ans. } f(x) = \begin{cases} \frac{d}{a}x, & 0 < x < a \\ \frac{d}{a-l}(x-l), & a < x < l \end{cases}$$

$$\text{Half period sine series } f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

...(1)

$$b_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l} dx = \frac{2}{l} \left[ \int_0^a \left( \frac{d}{a}x \right) \sin \frac{n\pi x}{l} dx + \int_a^l \frac{d}{a-l}(x-l) \sin \frac{n\pi x}{l} dx \right]$$

$$= \frac{2}{l} \left[ \frac{d}{a} \left\{ x \left( \frac{-\cos \frac{n\pi x}{l}}{n\pi/l} \right) - 1 \left( \frac{-\sin \frac{n\pi x}{l}}{n^2 \pi^2 / l^2} \right) \right\}_0^a + \frac{d}{(a-l)} \left\{ (x-l) \left( \frac{-\cos \frac{n\pi x}{l}}{n\pi/l} \right) - 1 \left( \frac{-\sin \frac{n\pi x}{l}}{n^2 \pi^2 / l^2} \right) \right\}_a^l \right]$$

$$= \frac{2}{l} \left[ \frac{d}{a} \left\{ \frac{-al}{n\pi} \cos \frac{n\pi a}{l} + \frac{l^2}{n^2 \pi^2} \sin \frac{n\pi a}{l} \right\} - \frac{d}{a-l} \left\{ -\frac{(a-l)l}{n\pi} \cos \frac{n\pi a}{l} + \frac{l^2}{n^2 \pi^2} \sin \frac{n\pi a}{l} \right\} \right]$$

$$= \frac{2}{l} \left[ -\frac{dl}{n\pi} \cos \frac{n\pi a}{l} + \frac{dl^2}{an^2 \pi^2} \sin \frac{n\pi a}{l} + \frac{dl}{n\pi} \cos \frac{n\pi a}{l} - \frac{dl^2}{(a-l)n^2 \pi^2} \sin \frac{n\pi a}{l} \right]$$

$$= \frac{2}{l} \frac{dl^2}{n^2 \pi^2} \left( \frac{1}{a} - \frac{1}{a-l} \right) \sin \frac{n\pi a}{l} = \frac{2dl}{n^2 \pi^2} \left( \frac{a-l-a}{a(a-l)} \right) \sin \frac{n\pi a}{l}$$

$$b_n = -\frac{2dl^2}{n^2\pi^2a(l-a)} \sin \frac{n\pi a}{l} = \frac{2dl^2}{n^2\pi^2a(l-a)} \sin \frac{n\pi a}{l}$$

Hence from (1),

$$f(x) = \sum_{n=1}^{\infty} \frac{2dl^2}{n^2\pi^2a(l-a)} \sin \frac{n\pi a}{l} \sin \frac{n\pi x}{l}$$

**Q. 6. (b) Find the Fourier series expansion for  $f(x) = x + \frac{x^2}{4}$ ,  $-\pi \leq x \leq \pi$ .**

**Ans.**  $f(x) = x + \frac{x^2}{4}$ ,  $-\pi \leq x \leq \pi$

Fourier series for function  $f(x)$  is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

...(1)

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} \left( x + \frac{x^2}{4} \right) dx = \frac{1}{\pi} \left[ \frac{x^2}{2} + \frac{x^3}{12} \right]_{-\pi}^{\pi} = \frac{1}{\pi} \left[ \left( \frac{\pi^2}{2} + \frac{\pi^3}{12} \right) - \left( \frac{\pi^2}{2} - \frac{\pi^3}{12} \right) \right] = \frac{1}{\pi} \left( \frac{2\pi^3}{12} \right) = \frac{\pi^2}{6}$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} \left( x + \frac{x^2}{4} \right) \cos nx \, dx \\ &= \frac{1}{\pi} \left[ \left( x + \frac{x^2}{4} \right) \left( \frac{\sin nx}{n} \right) - \left( 1 + \frac{2x}{4} \right) \left( -\frac{\cos nx}{n^2} \right) + \left( \frac{2}{4} \right) \left( -\frac{\sin nx}{n^3} \right) \right]_{-\pi}^{\pi} \\ &= \frac{1}{\pi} \left[ \left( 1 + \frac{2\pi}{4} \right) \frac{\cos n\pi}{n^2} - \left( 1 - \frac{2\pi}{4} \right) \frac{\cos n\pi}{n^2} \right] = \frac{1}{\pi} \left[ 1 + \frac{2\pi}{4} - 1 + \frac{2\pi}{4} \right] \frac{\cos n\pi}{n^2} = \frac{\cos n\pi}{n^2} = \frac{(-1)^n}{n^2} \end{aligned}$$

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} \left( x + \frac{x^2}{4} \right) \sin nx \, dx \\ &= \frac{1}{\pi} \left[ \left( x + \frac{x^2}{4} \right) \left( -\frac{\cos nx}{n} \right) - \left( 1 + \frac{2x}{4} \right) \left( -\frac{\sin nx}{n^2} \right) + \left( \frac{2}{4} \right) \left( \frac{\cos nx}{n^3} \right) \right]_{-\pi}^{\pi} \\ &= \frac{1}{\pi} \left[ \left\{ -\left( \pi + \frac{\pi^2}{4} \right) \frac{\cos n\pi}{n} + \frac{1 \cos n\pi}{2 n^3} \right\} - \left\{ -\left( -\pi + \frac{\pi^2}{4} \right) \frac{\cos n\pi}{n} + \frac{1 \cos n\pi}{2 n^3} \right\} \right] \\ &= \frac{1}{\pi} \left[ \left\{ -\pi - \frac{\pi^2}{4} \right\} \frac{\cos n\pi}{n} + \frac{1 \cos n\pi}{2 n^3} + \left\{ -\pi + \frac{\pi^2}{4} \right\} \frac{\cos n\pi}{n} - \frac{1 \cos n\pi}{2 n^3} \right] \\ &= \frac{1}{\pi} \left[ \left\{ -\pi - \frac{\pi^2}{4} - \pi + \frac{\pi^2}{4} \right\} \frac{\cos n\pi}{n} \right] \end{aligned}$$

$$b_n = \frac{1}{\pi} (-2\pi) \frac{\cos n\pi}{n}, \quad b_n = -\frac{2(-1)^n}{n} = \frac{2(-1)^{n+1}}{n}$$

$$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2} + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx$$

**Q. 6. (c) Solve the P.D.E**

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$$

**Ans.**  $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$

A.E. is  $m^2 - 2m + 1 = 0 \Rightarrow (m - 1)^2 = 0 \Rightarrow m = 1, 1$

C.F. =  $f_1(y + x) + x f_2(y + x)$

P.I. =  $\frac{1}{D^2 - 2DD' + D'^2} \sin x$

$D^2 = -1, DD' = 0, D'^2 = 0, = \frac{1}{-1 - 0 + 0} \sin x = -\sin x$

The complete solution is  $y = \text{C.F.} + \text{P.I.}$

$y = f_1(y + x) + x f_2(y + x) - \sin x$

**Q. 7. (a) Find the temperature in a bar of length 2 whose end are kept at zero and lateral surface is insulated if the initial temperature is  $\sin \frac{\pi x}{2} + 3 \sin 5\pi \frac{x}{2}$ .**

**Ans.**  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$  ... (1)

Boundary condition are

$u(0, t) = 0, u(2, t) = 0$

Initial condition is  $u(x, 0) = \sin \frac{\pi x}{2} + 3 \sin 5\pi \frac{x}{2}$

Let  $u = X(x) T(t)$  be the solution of equation (1)

(1)  $\Rightarrow \frac{T''}{c^2 T} = \frac{X''}{X} = \text{constant} = -p^2$

$\frac{X''}{X} = -p^2$

$X'' + p^2 X = 0$

$m^2 + p^2 = 0$

$m = \pm pi$

$X = c_1 \cos px + c_2 \sin px$

$u(x, t) = (c_1 \cos px + c_2 \sin px) c_3 e^{-c^2 p^2 t}$  ... (2)

using B. C.  $u(0, t) = 0$

$0 = c_1 c_3 e^{-c^2 p^2 t} \Rightarrow c_1 = 0$  ... (3)

Using B. C.  $u(2, t) = 0$

$0 = (c_1 \cos 2p + c_2 \sin 2p) c_3 e^{-c^2 p^2 t}$

$c_2 \sin 2p = 0 \Rightarrow \sin 2p = 0 = \sin n\pi$

$p = \frac{n\pi}{2}$  ... (4)

putting (3) & (4) in (2), we get

$$u(x, t) = c_2 c_3 \sin \frac{n\pi x}{2} e^{-\frac{n^2 \pi^2 c^2 t}{4}}$$

The general solution is

$$u(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2} e^{-\frac{n^2 \pi^2 c^2 t}{4}}$$

...(5)

using I.C in (5)

$$u(x, 0) = \sin \frac{\pi x}{2} + 3 \sin \frac{5\pi x}{2}$$

$$\sin \frac{\pi x}{2} + 3 \sin \frac{5\pi x}{2} = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2}$$

$$\Rightarrow b_1 = 1, n = 1 \text{ and } b_5 = 3, n = 5$$

Hence the required solution in

$$u(x, t) = \sin \frac{\pi x}{2} e^{-\frac{\pi^2 c^2 t}{4}} + 3 \sin \frac{5\pi x}{2} e^{-\frac{25\pi^2 c^2 t}{4}}$$

**Q. 7. (b) Apply the method of separation of variables to solve  $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ .**

$$\text{Ans. } \frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

Let  $Z = X(x) Y(y)$

...(1)

Where  $X$  is a function of  $x$  alone and  $Y$  that of  $y$  alone. Substituting this value of  $z$  in the given equation, we have

$$X'' Y - 2X' Y + XY' = 0, \text{ where } X' = \frac{dX}{dx}, Y' = \frac{dY}{dy}$$

separating the variables, we get

$$\frac{X'' - 2X'}{X} = -\frac{Y'}{Y} = \text{constant} = k$$

...(2)

$$\therefore \frac{X'' - 2X'}{X} = k \Rightarrow X'' - 2X' - kX = 0$$

...(3)

$$\text{and } \frac{-Y'}{Y} = k \Rightarrow Y' + kY = 0$$

...(4)

To solve equation (3), the auxiliary equation is

$$m^2 - 2m - k = 0 \Rightarrow m = 1 \pm \sqrt{1+k}$$

$$X = c_1 e^{(1+\sqrt{1+k})x} + c_2 e^{(1-\sqrt{1+k})x}$$

and the solution of (4) is  $Y = c_3 e^{-ky}$

from (1), we get

$$z = \left[ c_1 e^{(1+\sqrt{1+k})x} + c_2 e^{(1-\sqrt{1+k})x} \right] c_3 e^{-ky}, \quad z = [A e^{(1+\sqrt{1+k})x} + B e^{(1-\sqrt{1+k})x}] e^{-ky}$$

**Q. 7. (c) Solve the P.D.E by separation of variables method,**

$$u_{xx} = u_y + 2u, \quad u(0, y) = 0, \quad \frac{\partial}{\partial x} u(0, y) = 1 + e^{-3y}$$

$$\text{Ans. } u_{xx} = u_y + 2u \quad \dots(1)$$

$$u(0, y) = 0, \frac{\partial}{\partial x} u(0, y) = 1 + e^{-3y}$$

$$\text{Let } u(x, y) = X(x) Y(y) \quad \dots(2)$$

Substituting in the given equation (1), we have

$$X'' Y = X Y' + 2XY$$

$$\frac{X''}{X} - 2 = \frac{Y'}{Y} = k$$

$$X'' - 2X = kX \Rightarrow X'' - (2+k)X = 0$$

$$m^2 - (2+k) = 0$$

$$m = \pm \sqrt{2+k}$$

$$X = c_1 e^{x\sqrt{2+k}} + c_2 e^{-x\sqrt{2+k}}$$

$$\frac{Y'}{Y} = k \Rightarrow \log Y = ky + \log c_3$$

$$Y = c_3 e^{ky}$$

$$u(x, y) = \left[ c_1 e^{x\sqrt{2+k}} + c_2 e^{-x\sqrt{2+k}} \right] c_3 e^{ky}$$

$$u(x, y) = (A e^{x\sqrt{2+k}} + B e^{-x\sqrt{2+k}}) e^{ky} \quad \dots(3)$$

$$\frac{\partial u}{\partial x} = \left[ A e^{x\sqrt{2+k}} - B e^{-x\sqrt{2+k}} \right] \sqrt{2+k} e^{ky}$$

$$\text{using in equation (3), } u(0, y) = 0 \Rightarrow A + B = 0 \Rightarrow B = -A$$

$$\frac{\partial u}{\partial x}(0, y) = 1 + e^{-3y} \Rightarrow [A - B] \sqrt{2+k} e^{ky} = 1 + e^{-3y}$$

$$(A + B)(\sqrt{2+k}) e^{ky} = 1 + e^{-3y}$$

$$2A(\sqrt{2+k}) e^{ky} = 1 + e^{-3y}$$

$$\Rightarrow 2A\sqrt{2+k} = 1, k = 0$$

$$\text{and } 2A\sqrt{2+k} = 1, k = -3$$

$$\text{When } k = 0, A = \frac{1}{2\sqrt{2+0}} = \frac{1}{2\sqrt{2}} \text{ and } B = -\frac{1}{2\sqrt{2}}$$

$$\text{When } k = -3, A = \frac{1}{2\sqrt{2-3}} = \frac{1}{2i} \text{ and } B = \frac{1}{2i}$$

$$u(x, y) = \frac{1}{2\sqrt{2}} \left( e^{x\sqrt{2}} - e^{-x\sqrt{2}} \right) + \left[ \frac{e^{ix} - e^{-ix}}{2i} \right] e^{-3y}$$

$$= \frac{2 \sin h(x\sqrt{2})}{2\sqrt{2}} + \sin x \cdot e^{-3y}$$

$$u(x, y) = \frac{1}{\sqrt{2}} \sin h(x\sqrt{2}) + e^{-3y} \sin x$$